SL Paper 1

A function is represented by the equation

$$f(x) = ax^2 + \frac{4}{x} - 3$$

a. Find $f'(x)$. [3]

b. The function $f(x)$ has a local maximum at the point where $x = -1$. [3]

Find the value of *a*.

The function $f(x)$ is such that $f'(x) < 0$ for $1 < x < 4$. At the point $\mathrm{P}(4, 2)$ on the graph of $f(x)$ the gradient is zero.

a. Write down the equation of the tangent to the graph of $f(x)$ at P. [2]

b. State whether $f(4)$ is greater than, equal to or less than $f(2)$. [2]

c. Given that $f(x)$ is increasing for $4 \leqslant x < 7$, what can you say about the point P? [2]

Consider the function $f\left(x\right) = \frac{x^4}{4}$.

a. Find *f'*(*x*) [1]

b. Find the gradient of the graph of *f* at $x = -\frac{1}{2}$. [2]

c. Find the *x*-coordinate of the point at which the **normal** to the graph of *f* has gradient $-\frac{1}{8}$. [3]

Consider the curve $y = x^2$.

a. Write down $\frac{\mathrm{d}y}{\mathrm{d}x}$. [1]

b. The point $\mathrm{P}(3, 9)$ lies on the curve $y = x^2$. Find the gradient of the tangent to the curve at P . [2]

c. The point $\mathrm{P}(3, 9)$ lies on the curve $y = x^2$. Find the equation of the normal to the curve at P . Give your answer in the form $y = mx + c$. [3]

The equation of a curve is given as $y = 2x^2 - 5x + 4$.

a. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

b. The equation of the line *L* is $6x + 2y = -1$. [4]

Find the *x*-coordinate of the point on the curve $y = 2x^2 - 5x + 4$ where the tangent is parallel to *L*.

Consider the function $f(x) = ax^3 - 3x + 5$, where $a \neq 0$.

a. Find $f'(x)$. [2]

b. Write down the value of $f'(0)$. [1]

c. The function has a local maximum at *x* = –2. [3]

Calculate the value of *a*.

Let $f(x) = 2x^2 + x - 6$

a. Find $f'(x)$. [3]

b. Find the value of $f'(-3)$. [1]

c. Find the value of x for which $f'(x) = 0$. [2]

The coordinates of point A are $(6, \ -7)$ and the coordinates of point B are $(-6, \ 2)$. Point M is the midpoint of AB.

L_1 is the line through A and B.

The line L_2 is perpendicular to L_1 and passes through M.

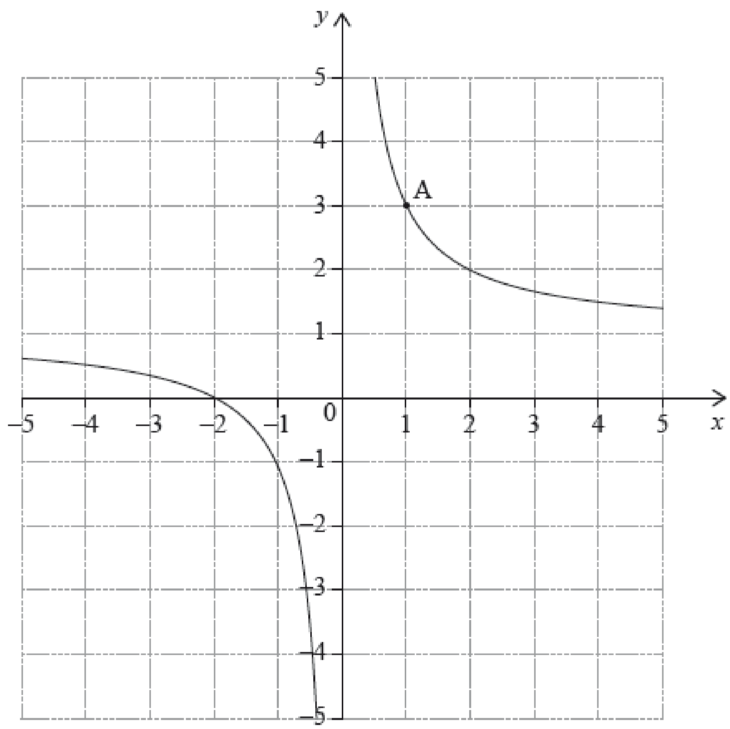
a. Find the coordinates of M. [2]

b. Find the gradient of L_1. [2]

c.i. Write down the gradient of L_2. [1]

c.ii. Write down, in the form $y = mx + c$, the equation of L_2. [1]

The diagram shows part of the graph of a function $y = f(x)$. The graph passes through point $\mathrm{A}(1,\ 3)$.



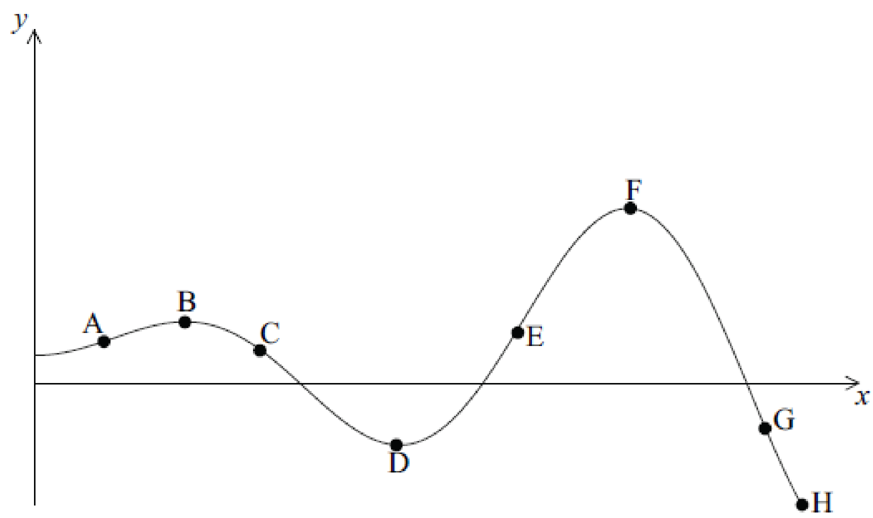
The tangent to the graph of $y = f(x)$ at A has equation $y = -2x + 5$. Let N be the normal to the graph of $y = f(x)$ at A.

a. Write down the value of $f(1)$. [1]

b. Find the equation of N. Give your answer in the form $ax + by + d = 0$ where $a,\ b,\ d \in \mathbb{Z}$. [3]

c. Draw the line N on the diagram above. [2]

Consider the graph of the function $y = f(x)$ defined below.



Write down **all** the labelled points on the curve

a. that are local maximum points; [1]

b. where the function attains its least value; [1]

c. where the function attains its greatest value; [1]

d. where the gradient of the tangent to the curve is positive; [1]

e. where $f(x) > 0$ and $f'(x) < 0$. [2]

Consider the function $f(x) = \frac{1}{2}x^3 - 2x^2 + 3$.

a. Find $f'(x)$. [2]

b. Find $f''(x)$. [2]

c. Find the equation of the tangent to the curve of f at the point $(1, 1.5)$. [2]

A cuboid has a rectangular base of width x cm and length $2x$ cm . The height of the cuboid is h cm . The total length of the edges of the cuboid is 72 cm.

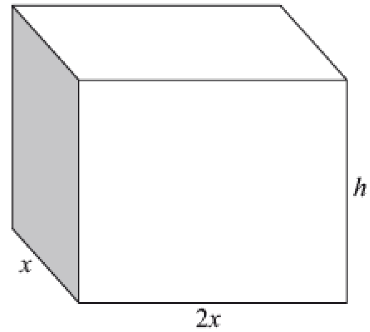


diagram not to scale

The volume, V, of the cuboid can be expressed as $V = ax^2 - 6x^3$.

a. Find the value of a. [3]

b. Find the value of x that makes the volume a maximum. [3]

The point A has coordinates (4 , –8) and the point B has coordinates (–2 , 4).

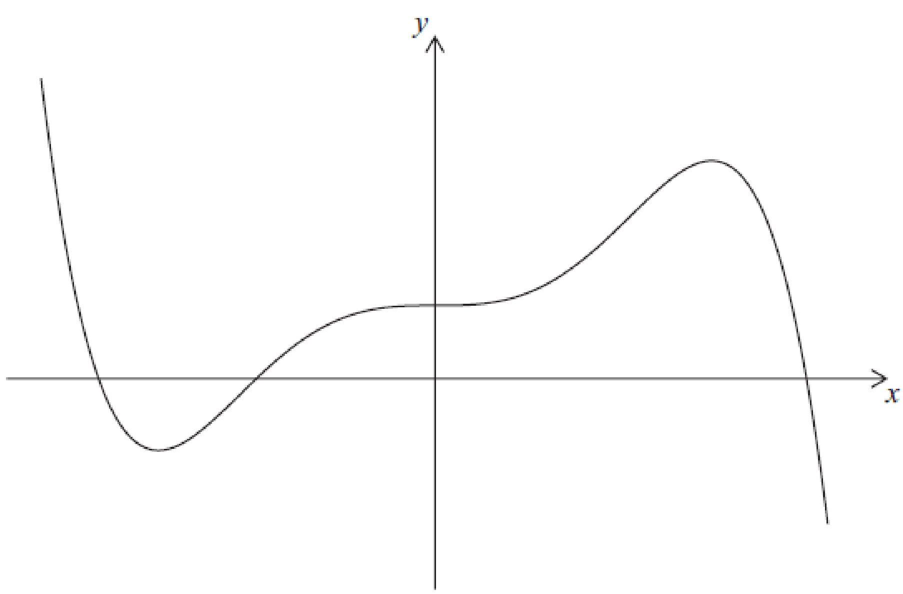
The point D has coordinates (–3 , 1).

a. Write down the coordinates of C, the midpoint of line segment AB. [2]

b. Find the gradient of the line DC. [2]

c. Find the equation of the line DC. Write your answer in the form *ax* + *by* + *d* = 0 where *a* , *b* and *d* are integers. [2]

A sketch of the function $f(x) = 5x^3 - 3x^5 + 1$ is shown for $-1.5 \leqslant x \leqslant 1.5$ and $-6 \leqslant y \leqslant 6$.



a. Write down $f'(x)$. [2]

b. Find the equation of the tangent to the graph of $y = f(x)$ at $(1, 3)$. [2]

c. Write down the coordinates of the second point where this tangent intersects the graph of $y = f(x)$. [2]

A quadratic function f is given by $f(x) = ax^2 + bx + c$. The points $(0,\ 5)$ and $(-4,\ 5)$ lie on the graph of $y = f(x)$.

The y-coordinate of the minimum of the graph is 3.

a. Find the equation of the axis of symmetry of the graph of $y = f(x)$. [2]

b. Write down the value of c. [1]

c. Find the value of a and of b. [3]

Consider the curve $y = x^2 + \frac{a}{x} - 1,\ x \neq 0$.

a. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

b. The gradient of the tangent to the curve is -14 when $x = 1$. [3]

Find the value of a.

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a. The equation of line L_1 is $y = 2.5x + k$. Point $\mathrm{A}\ (3,\ -2)$ lies on L_1. [2]

Find the value of k.

b. The line L_2 is perpendicular to L_1 and intersects L_1 at point A. [1]

Write down the gradient of L_2.

c. Find the equation of L_2. Give your answer in the form $y = mx + c$. [2]

d. Write your answer to part (c) in the form $ax + by + d = 0$ where a, b and $d \in \mathbb{Z}$. [1]

A small manufacturing company makes and sells x machines each month. The monthly cost C , in dollars, of making x machines is given by

$$C(x) = 2600 + 0.4x^2.$$

The monthly income I , in dollars, obtained by selling x machines is given by

$$I(x) = 150x - 0.6x^2.$$

$P(x)$ is the monthly profit obtained by selling x machines.

a. Find $P(x)$. [2]

b. Find the number of machines that should be made and sold each month to maximize $P(x)$. [2]

c. Use your answer to part (b) to find the selling price of **each machine** in order to maximize $P(x)$. [2]

The equation of line L_1 is $y = -\frac{2}{3}x - 2$.

Point P lies on L_1 and has x-coordinate -6.

The line L_2 is perpendicular to L_1 and intersects L_1 when $x = -6$.

a. Write down the gradient of L_1. [1]

b. Find the y-coordinate of P. [2]

c. Determine the equation of L_2. Give your answer in the form $ax + by + d = 0$, where a, b and d are integers. [3]

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a. Expand the expression $x(2x^3 - 1)$. [2]

b. Differentiate $f(x) = x(2x^3 - 1)$. [2]

c. Find the x-coordinate of the local minimum of the curve $y = f(x)$. [2]

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a. Consider the function $f(x) = ax^2 + c$. [1]

Find $f'(x)$

b. Point $\mathrm{A}(-2, 5)$ lies on the graph of $y = f(x)$. The gradient of the tangent to this graph at A is -6 . [3]

Find the value of a .

c. Find the value of c . [2]

Maria owns a cheese factory. The amount of cheese, in kilograms, Maria sells in one week, Q, is given by

$$Q = 882 - 45p,$$

where p is the price of a kilogram of cheese in euros (EUR).

Maria earns $(p - 6.80)$ EUR for each kilogram of cheese sold.

To calculate her weekly profit W, in EUR, Maria multiplies the amount of cheese she sells by the amount she earns per kilogram.

a. Write down how many kilograms of cheese Maria sells in one week if the price of a kilogram of cheese is 8 EUR. [1]

b. Find how much Maria earns in one week, from selling cheese, if the price of a kilogram of cheese is 8 EUR. [2]

c. Write down an expression for W in terms of p. [1]

d. Find the price, p, that will give Maria the highest weekly profit. [2]

Consider the curve $y = x^3 + kx$.

a. Write down $\frac{\mathrm{d}y}{\mathrm{d}x}$. [1]

b. The curve has a local minimum at the point where $x = 2$. [3]

Find the value of k.

c. The curve has a local minimum at the point where $x = 2$. [2]

Find the value of y at this local minimum.

$f(x) = 5x^3 - 4x^2 + x$

a. Find *f*'(*x*). [3]

b. Find using your answer to part (a) the *x*-coordinate of [3]

(i) the local maximum point;

(ii) the local minimum point.

a. The equation of the straight line L_1 is $y = 2x - 3$. [1]

Write down the y-intercept of L_1 .

b. Write down the gradient of L_1 . [1]

c. The line L_2 is parallel to L_1 and passes through the point $(0,\ 3)$. [1]

Write down the equation of L_2 .

d. The line L_3 is perpendicular to L_1 and passes through the point $(-2,\ 6)$. [1]

Write down the gradient of L_3.

e. Find the equation of L_3 . Give your answer in the form $ax + by + d = 0$, where a , b and d are integers. [2]

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a. A company sells fruit juices in cylindrical cans, each of which has a volume of $340\ \mathrm{cm}^3$. The surface area of a can is $A\ \mathrm{cm}^2$ and is given by the formula [3]

$A = 2\pi r^2 + \frac{680}{r}$,

where r is the radius of the can, in cm.

To reduce the cost of a can, its surface area must be minimized.

Find $\frac{\mathrm{d}A}{\mathrm{d}r}$

b. Calculate the value of r that minimizes the surface area of a can. [3]

Consider the function $f(x) = 2x^3 - 5x^2 + 3x + 1$.

a. Find $f'(x)$. [3]

b. Write down the value of $f'(2)$. [1]

c. Find the equation of the tangent to the curve of $y = f(x)$ at the point $(2, 3)$. [2]

The equation of a curve is $y = \frac{1}{2}x^4 - \frac{3}{2}x^2 + 7$.

The gradient of the tangent to the curve at a point P is -10.

a. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

b. Find the coordinates of P. [4]

The table given below describes the behaviour of $f'(x)$, the derivative function of $f(x)$, in the domain $-4 < x < 2$.

x	$f'(x)$
$-4 < x < -2$	< 0
-2	0
$-2 < x < 1$	> 0
1	0
$1 < x < 2$	> 0

a. State whether $f(0)$ is greater than, less than or equal to $f(-2)$. Give a reason for your answer. [2]

b. The point P(–2, 3) lies on the graph of $f(x)$. [2]

Write down the equation of the tangent to the graph of $f(x)$ at the point P.

c. The point P(–2, 3) lies on the graph of $f(x)$. [2]

From the information given about $f'(x)$, state whether the point (–2, 3) is a maximum, a minimum or neither. Give a reason for your answer.

The straight line, L, has equation $2y - 27x - 9 = 0$.

a. Find the gradient of L. [2]

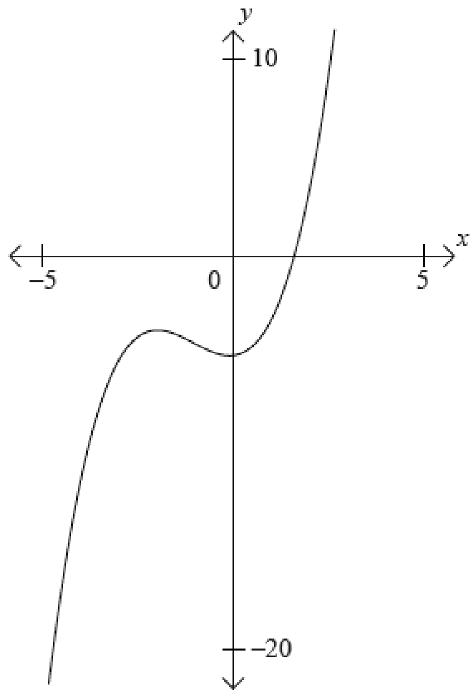
b. Sarah wishes to draw the tangent to $f(x) = x^4$ parallel to L. [1]

Write down $f'(x)$.

c, i.Find the x coordinate of the point at which the tangent must be drawn. [2]

c, ii.Write down the value of $f(x)$ at this point. [1]

Consider the graph of the function $f(x) = x^3 + 2x^2 - 5$.



a. Label the local maximum as A on the graph. [1]

b. Label the local minimum as B on the graph. [1]

c. Write down the interval where $f'(x) < 0$. [1]

d. Draw the tangent to the curve at $x = 1$ on the graph. [1]

e. Write down the equation of the tangent at $x = 1$. [2]

A curve is described by the function $f(x) = 3x - \frac{2}{x^2}, x \neq 0$.

a. Find $f'(x)$. [3]

b. The gradient of the curve at point A is 35. [3]

Find the *x*-coordinate of point A.

$$f(x) = \frac{1}{3}x^3 + 2x^2 - 12x + 3$$

a. Find $f'(x)$. [3]

b. Find the interval of x for which $f(x)$ is decreasing. [3]

Let $f(x) = x^4$.

a. Write down $f'(x)$. [1]

b. Point $\mathrm{P}(2, 6)$ lies on the graph of f. [2]

Find the gradient of the tangent to the graph of $y = f(x)$ at P.

c. Point $\mathrm{P}(2, 16)$ lies on the graph of f. [3]

Find the equation of the normal to the graph at P. Give your answer in the form $ax + by + d = 0$, where a, b and d are integers.

Consider $f : x \mapsto x^2 - 4$.

a. Find $f'(x)$. [1]

b. Let L be the line with equation $y = 3x + 2$. [1]

Write down the gradient of a line parallel to L.

c. Let L be the line with equation $y = 3x + 2$. [4]

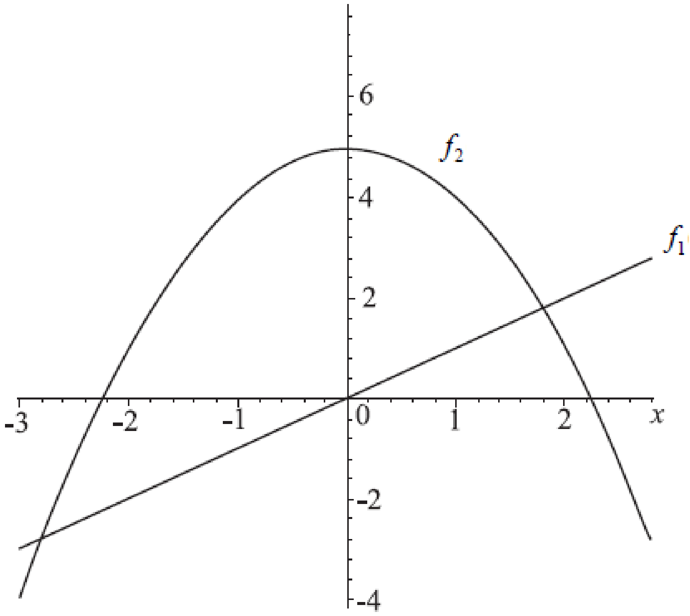
Let P be a point on the curve of f. At P, the tangent to the curve is parallel to L. Find the coordinates of P.

A function f is given by $f(x) = 4x^3 + \frac{3}{x^2} - 3,\ x \neq 0$.

a. Write down the derivative of f. [3]

b. Find the point on the graph of f at which the gradient of the tangent is equal to 6. [3]

The figure below shows the graphs of functions $f_1(x) = x$ and $f_2(x) = 5 - x^2$.



a. (i) Differentiate $f_1(x)$ with respect to *x*. [3]

(ii) Differentiate $f_2(x)$ with respect to *x*.

b. Calculate the value of *x* for which the gradient of the two graphs is the same. [2]

c. Draw the tangent to the **curved** graph for this value of *x* on the figure, showing clearly the property in part (b). [1]

A function is given as $f(x) = 2x^3 - 5x + \frac{4}{x} + 3, \ -5 \leqslant x \leqslant 10, \ x \neq 0.$

a. Write down the derivative of the function. [4]

b. Use your graphic display calculator to find the coordinates of the local minimum point of $f(x)$ in the given domain. [2]

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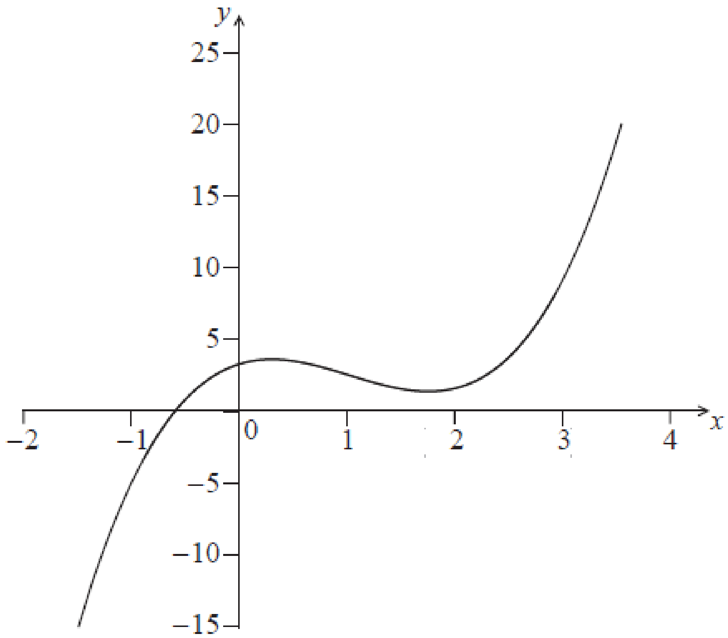
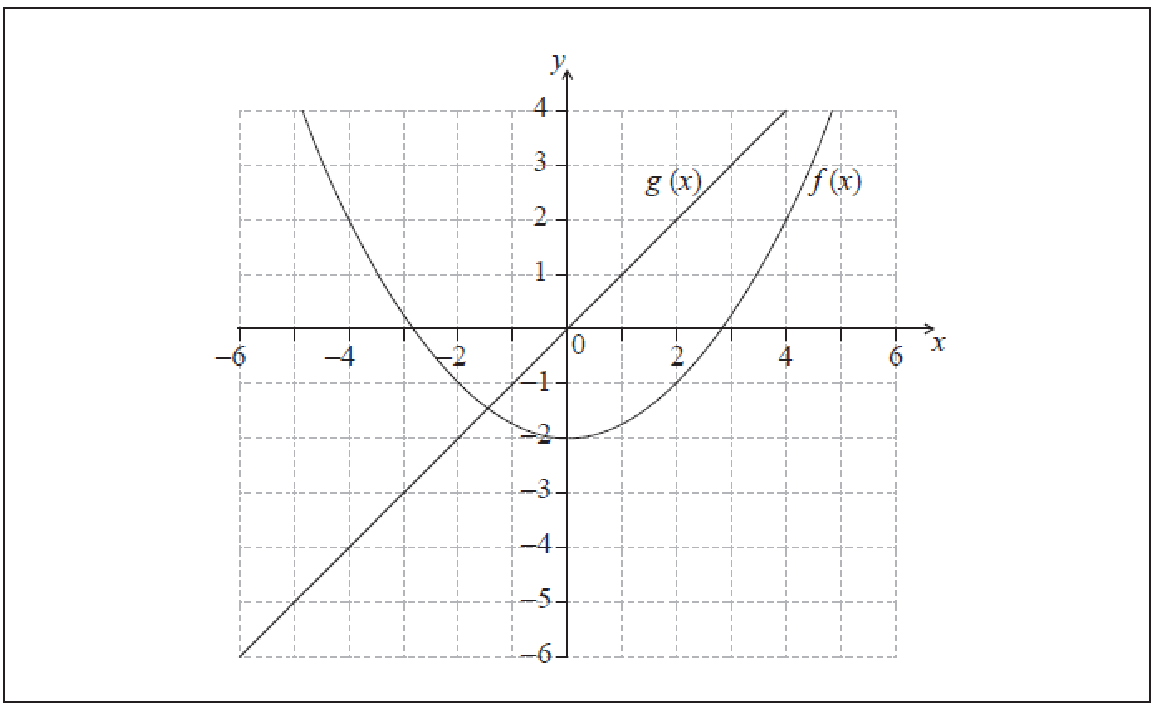
a. Consider the function $f(x) = x^3 - 3x^2 + 2x + 2$. Part of the graph of f is shown below. [3]



Find $f'(x)$.

b. There are two points at which the gradient of the graph of f is 11. Find the x-coordinates of these points. [3]

The figure shows the graphs of the functions $f(x) = \frac{1}{4}x^2 - 2$ and $g(x) = x$.



a. Differentiate $f(x)$ with respect to x . [1]

b. Differentiate $g(x)$ with respect to x . [1]

c. Calculate the value of x for which the gradients of the two graphs are the same. [2]

d. Draw the tangent to the parabola at the point with the value of x found in part (c). [2]

A factory produces shirts. The cost, C, in Fijian dollars (FJD), of producing x shirts can be modelled by

$$C(x) = (x - 75)^2 + 100.$$

The cost of production should not exceed 500 FJD. To do this the factory needs to produce at least 55 shirts and at most s shirts.

a. Find the cost of producing 70 shirts. [2]

b. Find the value of s. [2]

c. Find the number of shirts produced when the cost of production is lowest. [2]